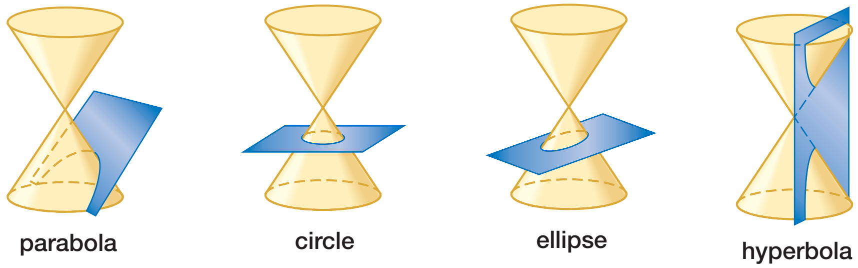
10-6 Conic Sections

Main Ideas

- Write equations of conic sections in standard form.
- Identify conic sections from their equations.

GET READY for the Lesson

Recall that parabolas, circles, ellipses, and hyperbolas are called *conic sections* because they are the cross sections formed when a double cone is sliced by a plane.



Standard Form The equation of any conic section can be written in the form of a general second-degree equation in two variables $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, where A, B, and C are not all zero. If you are given an equation in this general form, you may be able to complete the square to write the equation in one of the standard forms you have learned.

Reading Math

Ellipses In this lesson, the word *ellipse* means an ellipse that is not a circle.

CONCEPT SUMMARY — Standard Form of Conic Section

Conic Section	Standard Form of Equation
Parabola	$y = a(x - h)^2 + k$ or $x = a(y - k)^2 + h$
Circle	$(x - h)^2 + (y - k)^2 = r^2$
Ellipse	$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ or $\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$, $a \neq b$
Hyperbola	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ or $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$

EXAMPLE Rewrite an Equation of a Conic Section

1 **Write the equation $x^2 + 4y^2 - 6x - 7 = 0$ in standard form. State whether the graph of the equation is a *parabola, circle, ellipse,* or *hyperbola*. Then graph the equation.**

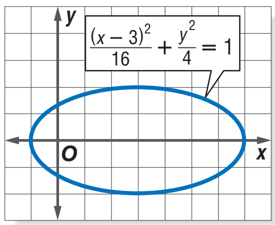
$x^2 + 4y^2 - 6x - 7 = 0$ Original equation

$x^2 - 6x + \blacksquare + 4y^2 = 7 + \blacksquare$ Isolate terms.

$x^2 - 6x + 9 + 4y^2 = 7 + 9$ Complete the square.

$(x - 3)^2 + 4y^2 = 16$ $x^2 - 6x + 9 = (x - 3)^2$

$\frac{(x-3)^2}{16} + \frac{y^2}{4} = 1$ Divide each side by 16.



The graph of the equation is an ellipse with its center at (3, 0).

CHECK Your Progress

1. Write the equation $x^2 + y^2 - 4x - 6y - 3 = 0$ in standard form. State whether the graph of the equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*. Then graph the equation.

Identify Conic Sections Instead of writing the equation in standard form, you can determine what type of conic section an equation of the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, where $B = 0$, represents by looking at A and C.

CONCEPT SUMMARY *Identifying Conic Sections*

Conic Section	Relationship of A and C
Parabola	$A = 0$ or $C = 0$, but not both.
Circle	$A = C$
Ellipse	A and C have the same sign and $A \neq C$.
Hyperbola	A and C have opposite signs.

EXAMPLE 2 Analyze an Equation of a Conic Section

Without writing the equation in standard form, state whether the graph of each equation is a *parabola, circle, ellipse,* or *hyperbola*.

a. $y^2 - 2x^2 - 4x - 4y - 4 = 0$

$A = -2$ and $C = 1$. Since A and C have opposite signs, the graph is a hyperbola.

b. $4x^2 + 4y^2 + 20x - 12y + 30 = 0$

$A = 4$ and $C = 4$. Since $A = C$, the graph is a circle.

c. $y^2 - 3x + 6y + 12 = 0$

$C = 1$. Since there is no x^2 term, $A = 0$. The graph is a parabola.

CHECK Your Progress

2A. $3x^2 + 3y^2 - 6x + 9y - 15 = 0$

2B. $4x^2 + 3y^2 + 12x - 9y + 14 = 0$

2C. $y^2 = 3x$

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CHECK Your Understanding

Example 1 (pp. 598–599)

Write each equation in standard form. State whether the graph of the equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*. Then graph the equation.

1. $y = x^2 + 3x + 1$
2. $y^2 - 2x^2 - 16 = 0$
3. $x^2 + y^2 = x + 2$
4. $x^2 + 4y^2 + 2x - 24y + 33 = 0$

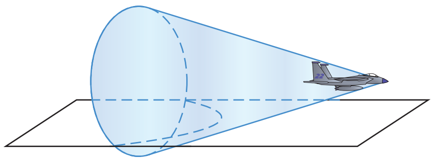
Example 2 (p. 599)

Without writing the equation in standard form, state whether the graph of each equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*.

5. $y^2 - x - 10y + 34 = 0$
6. $3x^2 + 2y^2 + 12x - 28y + 104 = 0$

AVIATION **For Exercises 7 and 8, use the following information.**

When an airplane flies faster than the speed of sound, it produces a shock wave in the shape of a cone. Suppose the shock wave generated by a jet intersects the ground in a curve that can be modeled by the equation $x^2 - 14x + 4 = 9y^2 - 36y$.



7. Identify the shape of the curve.

8. Graph the equation.

Exercises

HOMEWORK HELP	
For Exercises	See Examples
9–18	1
19–25	2

Write each equation in standard form. State whether the graph of the equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*. Then graph the equation.

9. $6x^2 + 6y^2 = 162$

10. $4x^2 + 2y^2 = 8$

11. $x^2 = 8y$

12. $4y^2 - x^2 + 4 = 0$

13. $(x - 1)^2 - 9(y - 4)^2 = 36$

14. $y + 4 = (x - 2)^2$

15. $(y - 4)^2 = 9(x - 4)$

16. $x^2 + y^2 + 4x - 6y = -4$

17. $x^2 + y^2 + 6y + 13 = 40$

18. $x^2 - y^2 + 8x = 16$

Without writing the equation in standard form, state whether the graph of each equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*.

19. $x^2 + y^2 - 8x - 6y + 5 = 0$

20. $3x^2 - 2y^2 + 32y - 134 = 0$

21. $y^2 + 18y - 2x = -84$

22. $7x^2 - 28x + 4y^2 + 8y = -4$

For Exercises 23–25, match each equation below with the situation that it could represent.

a. $9x^2 + 4y^2 - 36 = 0$

b. $0.004x^2 - x + y - 3 = 0$

c. $x^2 + y^2 - 20x + 30y - 75 = 0$

23. **SPORTS** the flight of a baseball

24. **PHOTOGRAPHY** the oval opening in a picture frame

25. **GEOGRAPHY** the set of all points that are 20 miles from a landmark

Real-World Career

Pilot

While flying the plane, a pilot must also be constantly scanning flight instruments, monitoring the engine, and communicating with the air traffic controller.

Math Online

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AVIATION **For Exercises 26–28, use the following information.**

A military jet performs for an air show. The path of the plane during one trick can be modeled by a conic section with equation $24x^2 + 1000y - 31{,}680x - 45{,}600 = 0$, where distances are represented in feet.

26. Identify the shape of the curved path of the jet. Write the equation in standard form.

27. If the jet begins its path upward or ascent at (0, 0), what is the horizontal distance traveled by the jet from the beginning of the ascent to the end of the descent?

28. What is the maximum height of the jet?

LIGHT **For Exercises 29 and 30, use the following information.**

A lamp standing near a wall throws an arc of light in the shape of a conic section. Suppose the edge of the light can be represented by the equation $3y^2 - 2y - 4x^2 + 2x - 8 = 0$.

29. Identify the shape of the edge of the light.

30. Graph the equation.

WATER **For Exercises 31 and 32, use the following information.**

If two stones are thrown into a lake at different points, the points of intersection of the resulting ripples will follow a conic section. Suppose the conic section has the equation $x^2 - 2y^2 - 2x - 5 = 0$.

31. Identify the shape of the curve.

32. Graph the equation.

Write each equation in standard form. State whether the graph of the equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*. Then graph the equation.

33. $x^2 + 2y^2 = 2x + 8$

34. $x^2 - 8y + y^2 + 11 = 0$

35. $9y^2 + 18y = 25x^2 + 216$

36. $3x^2 + 4y^2 + 8y = 8$

37. $x^2 + 4y^2 - 11 = 2(4y - x)$

38. $y + x^2 = -(8x + 23)$

39. $6x^2 - 24x - 5y^2 - 10y - 11 = 0$

40. $25y^2 + 9x^2 - 50y - 54x = 119$

Without writing the equation in standard form, state whether the graph of each equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*.

41. $5x^2 + 6x - 4y = x^2 - y^2 - 2x$

42. $2x^2 + 12x + 18 - y^2 = 3(2 - y^2) + 4y$

43. Identify the shape of the graph of the equation $2x^2 + 3x - 4y + 2 = 0$.

44. What type of conic section is represented by the equation $y^2 - 6y = x^2 - 8$?

EXTRA PRACTICE
See pages 913, 935.

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H.O.T. Problems

45. **OPEN ENDED** Write an equation of the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, where $A = 2$, that represents a circle.

46. **REASONING** Explain why the graph of the equation $x^2 + y^2 - 4x + 2y + 5 = 0$ is a single point.

CHALLENGE **For Exercises 47 and 48, use the following information.**

The graph of an equation of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$ is a special case of a hyperbola.

47. Identify the graph of such an equation.

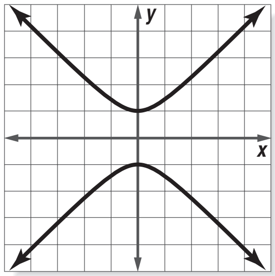
48. Explain how to obtain such a set of points by slicing a double cone with a plane.

49. **REASONING** Refer to Exercise 32 on page 587. Eccentricity can be studied for conic sections other than ellipses. The expression for the eccentricity of a hyperbola is $\frac{c}{a}$, just as for an ellipse. The eccentricity of a parabola is 1. Find inequalities for the eccentricities of noncircular ellipses and hyperbolas, respectively.

50. ***Writing in Math*** Use the information about conic sections on page 598 to explain how you can use a flashlight to make conic sections. Explain how you could point the flashlight at a ceiling or wall to make a circle and how you could point the flashlight to make a branch of a hyperbola.

STANDARDIZED TEST PRACTICE

51. ACT/SAT What is the equation of the graph?



A $y = x^2 + 1$ **B** $y - x = 1$

C $y^2 - x^2 = 1$ **D** $x^2 + y^2 = 1$

52. REVIEW The graph of $\left(\frac{x}{4}\right)^2 - \left(\frac{y}{5}\right)^2 = 1$ is a hyperbola. Which set of equations represents the asymptotes of the hyperbola's graph?

F $y = \frac{4}{5}x,\ y = -\frac{4}{5}x$

G $y = \frac{1}{4}x,\ y = -\frac{1}{4}x$

H $y = \frac{5}{4}x,\ y = -\frac{5}{4}x$

J $y = \frac{1}{5}x,\ y = -\frac{1}{5}x$

Spiral Review

Write an equation of the hyperbola that satisfies each set of conditions. (Lesson 10-5)

53. vertices (5, 10) and (5, –2), conjugate axis of length 8 units

54. vertices (6, –6) and (0, –6), foci $\left(3 \pm \sqrt{13}, -6\right)$

55. Find the coordinates of the center and foci and the lengths of the major and minor axes of the ellipse with equation $4x^2 + 9y^2 - 24x + 72y + 144 = 0$. Then graph the ellipse. (Lesson 10-4)

Simplify. Assume that no variable equals 0. (Lesson 6-1)

56. $(x^3)^4$

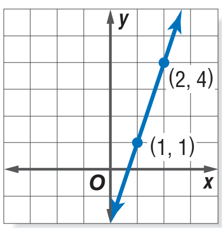
57. $(m^5n^{-3})^2m^2n^7$

58. $\dfrac{x^2y^{-3}}{x^{-5}y}$

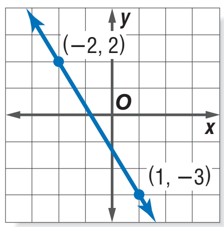
59. HEALTH The prediction equation $y = 205 - 0.5x$ relates a person's maximum heart rate for exercise y and age x. Use the equation to find the maximum heart rate for an 18-year old. (Lesson 2-5)

Write an equation in slope-intercept form for each graph. (Lesson 2-4)

60.



61.



GET READY for the Next Lesson

PREREQUISITE SKILL Solve each system of equations. (Lesson 3-2)

62. $y = x + 4$
$2x + y = 10$

63. $4x + y = 14$
$4x - y = 10$

64. $x + 5y = 10$
$3x - 2y = -4$